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Journal of Sound and Vibration 298 (2006) 194–205
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

FAT-based adaptive sliding control for flexible arms: Theory and experiments

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Received 7 January 2005; received in revised form 8 May 2006; accepted 11 May 2006
Available online 1 August 2006

Abstract

An adaptive sliding controller is proposed in this paper for flexible arms containing time-varying uncertainties with unknown bounds based on function approximation technique (FAT). The uncertainties are firstly represented by finite linear combinations of orthonormal basis with some unknown constant weighting vectors. Output error dynamics can thus be derived as a stable first-order filter driven by parameter error vectors. Appropriate update laws for the weighting vectors are selected using the Lyapunov method so that asymptotic convergence of the output error can be proved as long as a sufficient number of basis functions are used. Effects of the approximation error on system performance are also investigated in this paper. Both computer simulation and experimental results confirm the feasibility of the proposed control strategy.

1. Introduction

Conventional industrial robots are mostly designed with heavy links to reduce vibration and increase motion accuracy. Although this simplifies controller design, heavy links imply slower response and higher energy consumption. In recent years, development of lightweight robot manipulators has attracted much attention in the literature. With reduced weight, we have to consider flexibility of the links. Since a flexible link has infinitive number of vibration modes, it is impossible to feedback all of them to have a stabilizing controller. A feasible approach is to consider only a few significant modes and treat all others as uncertainties. Therefore, to design a practical controller for a flexible arm, these uncertainties should be carefully considered.

Several robust designs have been proposed for dealing with uncertainties and disturbances in a flexible manipulator. Gokasan et al. [1] implemented a DSP-based sliding controller for a flexible arm to achieve robust performance against unmodeled dynamics and parameter variations. Chen et al. [2] and Chen and Fukuda [3] regarded higher-order vibration modes and friction forces as bounded disturbances and a sliding controller was introduced to stabilize the system. Karandikar and Bandyopadhyay [4] used a sliding controller to drive a flexible arm to the vicinity of the target position and then a controller based on the linearized model

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0022-460X/$ - see front matter © 2006 Published by Elsevier Ltd.
doi:10.1016/j.jsv.2006.05.021

was applied to damp the vibration. Kwok and Lee [5] suggested to impose a fuzzy-like weighting factor on the sliding-based control signal such that the tracking control component has higher priority when the output error is large, and suppression of vibration is effective when the tracking error is small. Gu and Asokanthan [6] designed a hybrid controller based on the Lyapunov approach to generate control actions to the hub motor and distributed piezoelectric film actuator for a flexible arm. Hissenine and Lohmann [7] implemented two robust controllers: one is based on the sliding control and the other is on the nonlinear H_∞ theory. It was concluded that the later method suffered from computational complexity, although closed-loop stability as well as disturbance attenuation could be achieved. A robust LQ controller was proposed by Ryu et al. [8] for dealing with a large time-varying payload and parameter uncertainties. The system was formulated in the descriptor form with additive uncertainties in polytopic representation. The optimal robust performance was obtained based on the LMI theory.

Various adaptive strategies are available for controlling uncertain flexible arms. Bai et al. [9] presented an adaptive augmented state feedback controller for the tracking control for a two-link flexible robot. Feliu et al. [10] proposed an adaptive scheme for a flexible robot with uncertain payload. Instead of calculating the entire transfer function of the system, their method required only the estimation of the tip load but with acceleration feedback. Queiroz et al. [11] suggested an adaptive nonlinear boundary controller consisting of a boundary control torque applied to the hub and a boundary control force at the tip. A shear force sensor, however, was needed to be installed at the free end. Rokui and Khorasani [12] used output redefinition technique to obtain stable internal dynamics for a flexible arm and then an indirect adaptive linearizing controller was developed for updating unknown payload. Cheong et al. [13] separated the system dynamics into rigid and flexible parts using bandwidth modulation approach. An adaptive controller was designed for the flexible subsystem to arrive at fast vibration suppression. An adaptive energy-based robust controller was designed for multi-line flexible robots in Ref. [14]. Desired performance was achieved with automatic gain tuning in some controller component. Direct and indirect adaptive command shaping designs for flexible arms were analyzed in Ref. [15]. The direct approach was shown to be less sensitive to the noise effect on the performance.

Similar to most conventional robust designs, all robust approaches reviewed above have a common assumption that all uncertainties should be defined in some known bounded sets. On the other hand, similar to most adaptive schemes, all adaptive approaches mentioned above require that uncertain parameters be time invariant or slowly time-varying. Because lower frequency vibration modes are used to approximate system dynamics, it is generally not easy to know exact variation bounds of the neglected dynamics; therefore, robust designs are hard to apply. Since these uncertainties are time-varying, implementation of most adaptive controls is not easy either. In this paper, we would like to design an adaptive controller for flexible arms containing time-varying uncertainties with unknown bounds using function approximation technique (FAT) [16–22]. These uncertainties are firstly represented by finite linear combinations of orthonormal basis with some unknown constant weighting vectors. Output error dynamics can thus be derived as a stable first-order filter driven by parameter error vectors. Appropriate update laws for the weighting vectors can be selected so that the time derivative of some Lyapunov function candidate can be proved to be negative semi-definite. Effects of the approximation error on the system performance can also be investigated. To proof the effectiveness of the proposed scheme, both computer simulations and experiments are conducted.

This paper is organized as follows: Section 2 presents a discretized model of the flexible arm. An adaptive controller is designed in Section 3 with rigorous proof of closed-loop stability. Section 4 introduces experimental setup and presents results of computer simulations and experiments. Section 5 concludes this paper.

2. System model of a single-link flexible arm

The system under consideration is illustrated in Fig. 1. The Euler–Bernoulli beam is clamped to an actuator hub which can rotate in horizontal plane. The payload at the free end is modeled as a concentrated mass m. For an angular displacement $\theta(t)$ of the actuator hub at time $t \geqslant 0$, the arm will have a displacement $\omega(x,t)$ at position x with respect to the (X, Y) coordinate rotating with the hub. Therefore, the motion of a point along

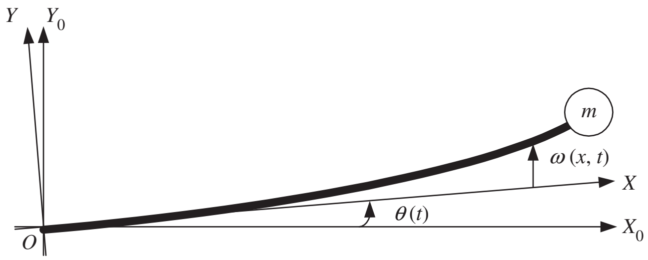

Fig. 1. Single-link flexible arm.

the beam is given by

$$\xi(x,t) = x\theta(t) + \omega(x,t). \tag{1}$$

It is well-known that using the Lagrange's method, the flexible arm can be described by a partial differential equation, and the deflection of the arm can be expressed as a weighted combination of mode shapes. By using the assumed mode method, the system can be approximated by a discretized model with proper accuracy into a convenient form for controller design [23,24,30]

$$\mathbf{M}(\mathbf{x})\ddot{\mathbf{x}} + \mathbf{f}(\mathbf{x}) = \mathbf{b}u, \tag{2}$$

where $\mathbf{x} = [\theta q_1 \ \ldots \ q_n]^{\mathrm{T}}$ and q_1 to q_n are flexible modes. If the first two modes are considered, the state vector becomes $\mathbf{x} = [\theta q_1 \ \ldots \ q_2]^{\mathrm{T}}$. Let $\phi_i(x)$ be the ith mode shape function, ρ the mass density and l the arm length, then the inertial matrix $\mathbf{M}(\mathbf{x})$ in Eq. (2) for $n = 2$ case can be represented as

$$\mathbf{M}(\mathbf{x}) = \begin{bmatrix} m_{01} + ml\phi_1 & m_{11} + m\phi_1\phi_1 & m_{12} + m\phi_1\phi_2 \\ m_{02} + ml\phi_2 & m_{21} + m\phi_2\phi_1 & m_{22} + m\phi_2\phi_2 \\ m_{00} + ml & m_{01} + ml\phi_1 & m_{02} + ml\phi_2 \end{bmatrix},$$

where

$$m_{00} = \int_0^l \rho x^2 \, \mathrm{d}x, \quad m_{0i} = \int_0^l \rho x \phi_i \, \mathrm{d}x \quad \text{and} \quad m_{ij} = \int_0^l \rho \phi_i \phi_j \, \mathrm{d}x.$$

The vector $\mathbf{f}(\mathbf{x})$ in Eq. (2) represents the lumped effects of centrifugal, Coriolis, damping, stiffness, friction and gravitational forces. It can be shown to be in the form

$$\mathbf{f}(\mathbf{x}) = [k_{11}q_1 + k_{22}q_2 \quad k_{21}q_1 + k_{22}q_2 \quad 0]^{\mathrm{T}},$$

where $k_{ij} = \int_0^l EI\phi_i''\phi_j'' \, \mathrm{d}x$. The vector $\mathbf{b} = [0\,0\,1]^{\mathrm{T}}$ is the gain for the applied torque u from the control motor. With this representation, the system inevitably contains model imprecision due to discretization. In practical implementation, it is in general very costly to improve model accuracy. Under this circumstance, a reasonable assumption is to regard $\mathbf{M}(\mathbf{x})$, $\mathbf{f}(\mathbf{x})$ and $\mathbf{b}$ as time-varying uncertainties with unknown variation bounds. Since most traditional robust designs require that system uncertainties be defined in known compact sets, they are not suitable to the present system. On the other hand, conventional adaptive schemes need the uncertain parameters be constant or slowly time varying, they are not applicable either. In this paper, we would like to design an FAT-based controller to deal with the problem with rigorous proof and experiments.

3. Controller design

In this section, we would like to derive an adaptive sliding controller for the uncertain flexible arm derived in the previous section based on FAT. We firstly ignore the approximation error by assuming a sufficient number of orthonormal functions are used. Under this condition, the output error can be proved to have asymptotic convergence performance. Afterwards the approximation error is considered, and boundedness of the output error can be concluded. The bound is proved to be a function of the approximation error. Finally, we consider the case when the approximation error bound is available. By including a robust term in the control input, asymptotic convergence of the output error can again be derived.

According to Kanoh and Lee [25], Lin and Chu [26], Ohta et al. [27] and Chen et al. [2], we may assume that the tip position of the arm is able to be approximated as a linear combination of the states in Eq. (2). Therefore, let us define the output function $y = \mathbf{c}^{\mathrm{T}}\mathbf{x}$ where $\mathbf{c} \in \Re^3$ is a known constant vector. Therefore, the output error can be denoted as $e = y - y_{\mathrm{d}}$, where y_{d} represents the desired output. Define a sliding surface as

$$s = \dot{e} + \lambda e = 0. \tag{3}$$

The positive constant λ determines the convergent rate of the output error on the sliding surface. Since in general $s(0) \neq 0$, we would like to design a controller so that $s = 0$ is attractive. Let us consider the dynamics of $s(t)$ for all $t \geqslant 0$ as

$$\dot{s} = (\mathbf{c}^{\mathrm{T}}\ddot{\mathbf{x}} - \ddot{y}_{\mathrm{d}}) + \lambda\dot{e}. \tag{4}$$

Using Eq. (2), the above equation can be rewritten as

$$\dot{s} = f + (g - \hat{g})u + \hat{g}u - \ddot{y}_{\mathrm{d}} + \lambda\dot{e}, \tag{5}$$

where $f = -\mathbf{c}^{\mathrm{T}}\mathbf{M}^{-1}\mathbf{F}$, $g = \mathbf{c}^{\mathrm{T}}\mathbf{M}^{-1}\mathbf{b}$ are unknown time functions and $\hat{g}$ is an estimate of g. Note that the inertia matrix is nonsingular for all $t \geqslant 0$. If an estimate of f is also available and $\hat{g}$ is nonsingular for all $t \geqslant 0$, then a controller can be designed to be

$$u = \frac{1}{\hat{g}}(-\hat{f} + \ddot{y}_d - \lambda\dot{e} - \eta s), \quad \eta > 0. \tag{6}$$

In practical implementation, some projection modification should be applied [28,29] to avoid singularity of $\hat{g}$. With this controller, Eq. (5) becomes

$$\dot{s} = -\eta s + (f - \hat{f}) + (g - \hat{g})u. \tag{7}$$

This implies that s is an output of a stable first-order filter driven by approximation errors of f and g. If some proper update laws can be found so that $\hat{f} \to f$ and $\hat{g} \to g$, then $e \to 0$ can be concluded from Eqs. (7) and (3). Since f and g are functions of time, traditional adaptive controllers are not applicable. On the other hand, since their variation bounds are not given, robust designs are not feasible either. Here, we would like to use FAT [16–20] to represent f, g, $\hat{f}$ and $\hat{g}$ as

$$f = \mathbf{w}_f^{\mathrm{T}}\mathbf{z}_f, \tag{8a}$$

$$g = \mathbf{w}_g^{\mathrm{T}}\mathbf{z}_g, \tag{8b}$$

$$\hat{f} = \hat{\mathbf{w}}_f^{\mathrm{T}}\mathbf{z}_f, \tag{8c}$$

$$\hat{g} = \hat{\mathbf{w}}_g^{\mathrm{T}}\mathbf{z}_g, \tag{8d}$$

where $\mathbf{w}_f \in \Re^{\beta_f \times 1}$ and $\mathbf{w}_g \in \Re^{\beta_g \times 1}$ are weighting vectors and $\mathbf{z}_f \in \Re^{\beta_f \times 1}$ and $\mathbf{z}_g \in \Re^{\beta_g \times 1}$ are matrices of basis functions. The number $\beta_{(\cdot)}$ represents the number of basis functions used. Effects of the approximation error will be considered later in this paper. Therefore, Eq. (7) can be rewritten as

$$\dot{s} = -\eta s + \tilde{\mathbf{w}}_f^{\mathrm{T}}\mathbf{z}_f + \tilde{\mathbf{w}}_g^{\mathrm{T}}\mathbf{z}_g u, \tag{9}$$

where $\tilde{\mathbf{w}}_f = \mathbf{w}_f - \hat{\mathbf{w}}_f$ and $\tilde{\mathbf{w}}_g = \mathbf{w}_g - \hat{\mathbf{w}}_g$. Since $\mathbf{w}_f$ and $\mathbf{w}_g$ are constant vectors, their update laws can be easily found by proper selection of a Lyapunov function. Let us consider a candidate as

$$V = \tfrac{1}{2}s^2 + \tfrac{1}{2}\tilde{\mathbf{w}}_f^{\mathrm{T}}\mathbf{Q}_f\tilde{\mathbf{w}}_f + \tfrac{1}{2}\tilde{\mathbf{w}}_g^{\mathrm{T}}\mathbf{Q}_g\tilde{\mathbf{w}}_g, \tag{10}$$

where $\mathbf{Q}_f$ and $\mathbf{Q}_g$ are positive definite matrices with proper dimensions. Taking the time derivative of V along the trajectory of Eq. (9), we have

$$\begin{aligned}\dot{V} &= s(-\eta s + \tilde{\mathbf{w}}_f^{\mathrm{T}}\mathbf{z}_f + \tilde{\mathbf{w}}_g^{\mathrm{T}}\mathbf{z}_g u) - \tilde{\mathbf{w}}_f^{\mathrm{T}}\mathbf{Q}_f\dot{\hat{\mathbf{w}}}_f - \tilde{\mathbf{w}}_g^{\mathrm{T}}\mathbf{Q}_g\dot{\hat{\mathbf{w}}}_g \\ &= -\eta s^2 + \tilde{\mathbf{w}}_f^{\mathrm{T}}(\mathbf{z}_f s - \mathbf{Q}_f\dot{\hat{\mathbf{w}}}_f) + \tilde{\mathbf{w}}_g^{\mathrm{T}}(\mathbf{z}_g s u - \mathbf{Q}_g\dot{\hat{\mathbf{w}}}_g).\end{aligned} \tag{11}$$

If we select the following update laws:

$$\begin{aligned}\dot{\hat{\mathbf{w}}}_f &= \mathbf{Q}_f^{-1}\mathbf{z}_f s, \\ \dot{\hat{\mathbf{w}}}_g &= \mathbf{Q}_g^{-1}\mathbf{z}_g s u,\end{aligned} \tag{12}$$

then Eq. (11) becomes

$$\dot{V} = -\eta s^2 \leqslant 0. \tag{13}$$

Hence $s, \tilde{\mathbf{w}}_f, \tilde{\mathbf{w}}_g \in L_\infty$. Square integrability of s is proved by

$$\int_0^\infty s^2\,\mathrm{d}t = -\eta^{-1}\int_0^\infty \dot{V}\,\mathrm{d}t = \eta^{-1}(V_0 - V_\infty) < \infty.$$

Boundedness of $\dot{s}$ can be easily justified from Eq. (9). Therefore, asymptotic convergence of s can be concluded by the Barbalat's lemma [31,32]. This further implies convergence of the output error e. Convergence of the parameters can be proved to depend on the PE condition of the reference input.

3.1. Consideration of approximation errors

In the above derivation, we assume that a sufficient number of orthonormal functions are used and the approximation error is ignored. Here, let us consider the effect of the approximation error on the performance of the closed-loop system. Instead of Eqs. (8a) and (8b), f and g can be represented as

$$f = \mathbf{w}_f^{\mathrm{T}}\mathbf{z}_f + \varepsilon_f, \tag{14a}$$

$$g = \mathbf{w}_g^{\mathrm{T}}\mathbf{z}_g + \varepsilon_g, \tag{14b}$$

where ε_f and ε_g are approximation errors of f and g, respectively. Hence, the error dynamics (9) can be rewritten to be

$$\dot{s} = -\eta s + \tilde{\mathbf{w}}_f^{\mathrm{T}}\mathbf{z}_f + \tilde{\mathbf{w}}_g^{\mathrm{T}}\mathbf{z}_g u + \varepsilon, \tag{15}$$

where $\varepsilon = \varepsilon(\varepsilon_f, \varepsilon_g, s, \ddot{y}_\mathrm{d})$ is a lumped approximation error. If we still select Eq. (12) as the update laws, then the time derivative of V in Eq. (10) along the trajectory of Eq. (15) can be computed to be

$$\dot{V} = -\eta s^2 + s\varepsilon. \tag{16}$$

Due to the existence of ε, we may not determine definiteness of $\dot{V}$ to conclude any stability property of the closed-loop system. Let us proceed by representing Eq. (16) in the form

$$\dot{V} \leqslant (-\eta|s| + |\varepsilon|)|s|.$$

If we choose a proper $\eta > 0$ and a suitable set of orthonormal functions, then $\dot{V} \leqslant 0$ whenever

$$s \in \left\{\sigma \,\middle|\, |\sigma| > \frac{|\varepsilon|}{\eta}\right\}.$$

This implies that the output error is bounded and the bound is a function of the approximation error.

3.2. The case when the approximation error bound is available

A usual assumption in the neural network-based control strategies [28] is that the bound of the approximation error is available, i.e., there exists a constant $\delta > 0$ such that $|\varepsilon| \leqslant \delta$. Suppose this assumption is also valid for the scheme derived above. The controller (6) can thus be modified to cover the effect of this bounded approximation error as

$$u = \frac{1}{\hat{g}}(-\hat{f} + \ddot{y}_{\mathrm{d}} - \lambda\dot{e} - \eta s + u_{\mathrm{robust}}). \tag{17}$$

The robust term u_{robust} is designed below. Let us consider the Lyapunov function candidate (10) again. Its time derivative is computed as

$$\begin{aligned}\dot{V} &= -\eta s^2 + s\varepsilon + s u_{\mathrm{robust}}, \\ &\leqslant -\eta s^2 + \delta|s| + s u_{\mathrm{robust}}.\end{aligned}$$

By selecting $u_{\mathrm{robust}} = -\delta\,\mathrm{sgn}(s)$, we may have $\dot{V} = -\eta s^2 \leqslant 0$, and asymptotic convergence of the output error can be further proved according to the Barbalat's lemma.

4. Simulation and experimental results

To verify the effectiveness of the proposed control strategy, both computer simulations and experiments are performed. In Section 4.1, the experimental setup is introduced. In Section 4.2, simulation results are presented. Experimental results are shown in Section 4.3.

4.1. Experiment setup

The proposed controller is implemented on an experimental system shown in Fig. 2. The flexible arm is constructed from a 0.55 m long and $0.6 \times 45\,\mathrm{mm}^2$ section spring steel strip. The net weight of the strip is 0.128 kg, and the mass per unit length is 0.233 kg/m. The mass of the payload is 0.025 kg. With this design, the arm is very flexible in the horizontal plane. What is worse is that the payload and the mass distribution are assumed to be unavailable; the controller design problem becomes extremely difficult. The actuator hub is directly coupled to a 90 W DC servo motor driven by a PWM driver. The hub angle and angular velocity are obtained from a 1024-pulse encoder installed on the motor shaft. An HCTL-2020 chip is used to sample the encoder reading. A 12-bit D/A and two 8-bit A/D channels are used to interface with a 133 MHz Pentium PC. Two strain gauges are employed to feedback system states. Strain gauge positions and their calibration are found in Ref. [26]. The proposed controller and update laws are implemented in a timer interrupt service routine under 10 ms sampling rate in the MS-DOS environment.

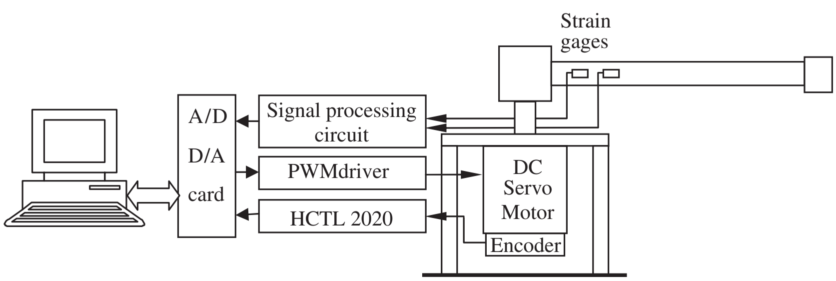

Fig. 2. Experimental setup.

4.2. Simulation results

Controller (6) is implemented with the update laws in Eq. (12) to control the given system. Initial condition for the simulation is $\mathbf{x}(0) = [-1\,0\,0]^T$ and we would like the end-point to track the trajectory

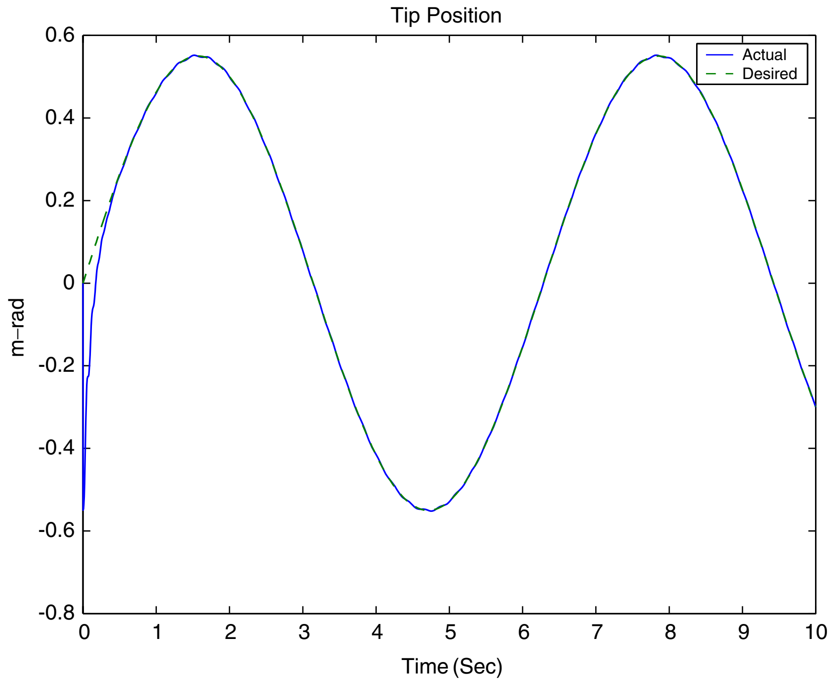

Fig. 3. Output tracking performance in simulation.

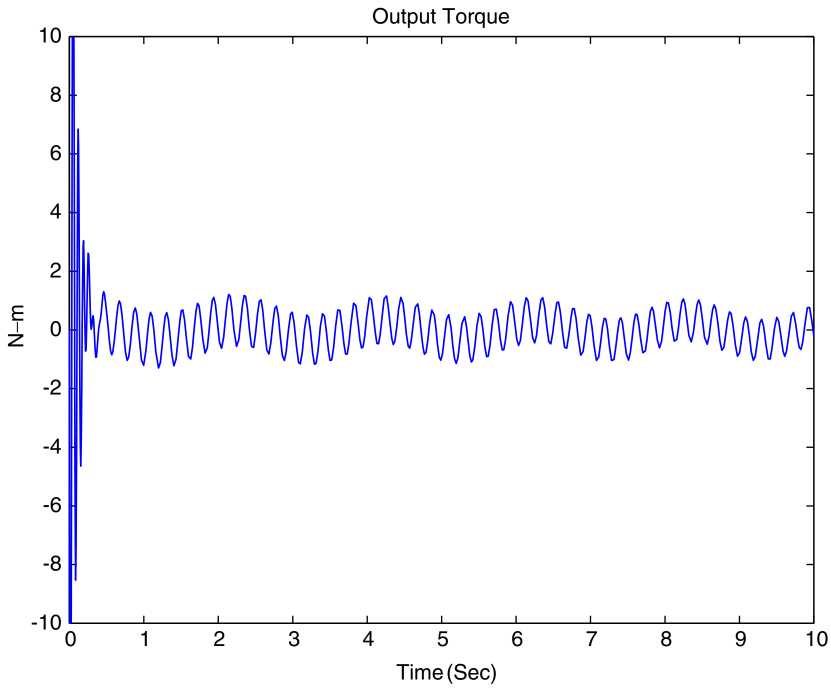

Fig. 4. Control torque in simulation.

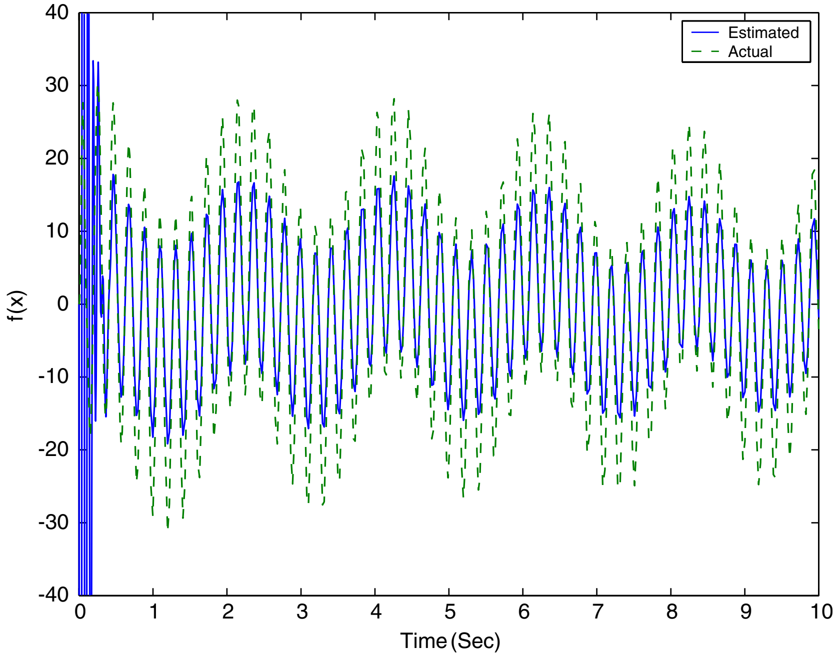

Fig. 5. Approximation of *f*(*x*) in simulation.

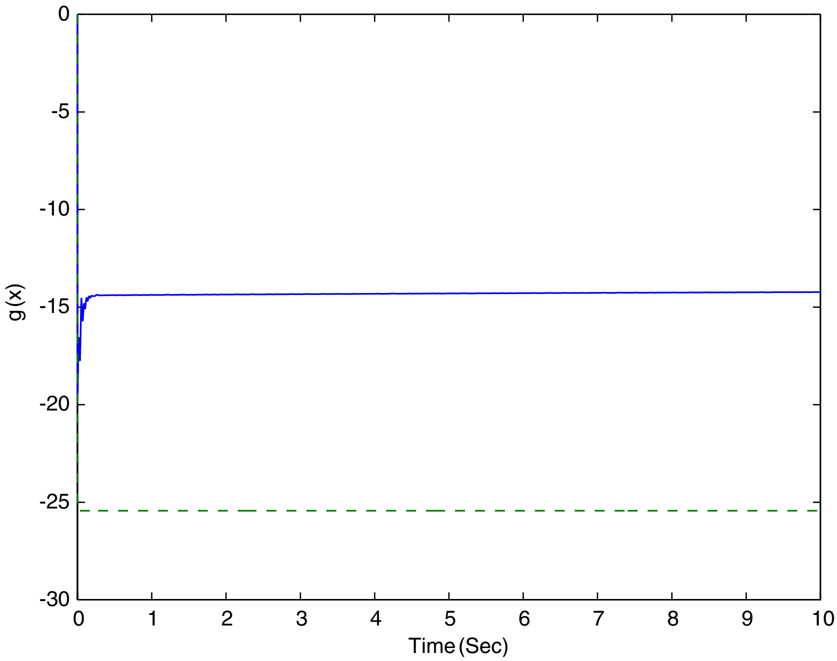

Fig. 6. Approximation of $g(x)$ in simulation.

$y_{\mathrm{d}} = 0.55 \sin t \, \mathrm{m} \cdot \mathrm{rad}$. The controller parameters are selected as $\lambda = 10$ and $\eta = 10$. Both f and g are approximated by the first nine terms of the Fourier series, respectively. Initial weighting vectors are chosen as

$$\hat{\mathbf{w}}_f(0) = [0 \;\; 0 \;\; \ldots \;\; 0]^{\mathrm{T}},$$
$$\hat{\mathbf{w}}_g(0) = [-20 \;\; 0 \;\; \ldots \;\; 0]^{\mathrm{T}}.$$

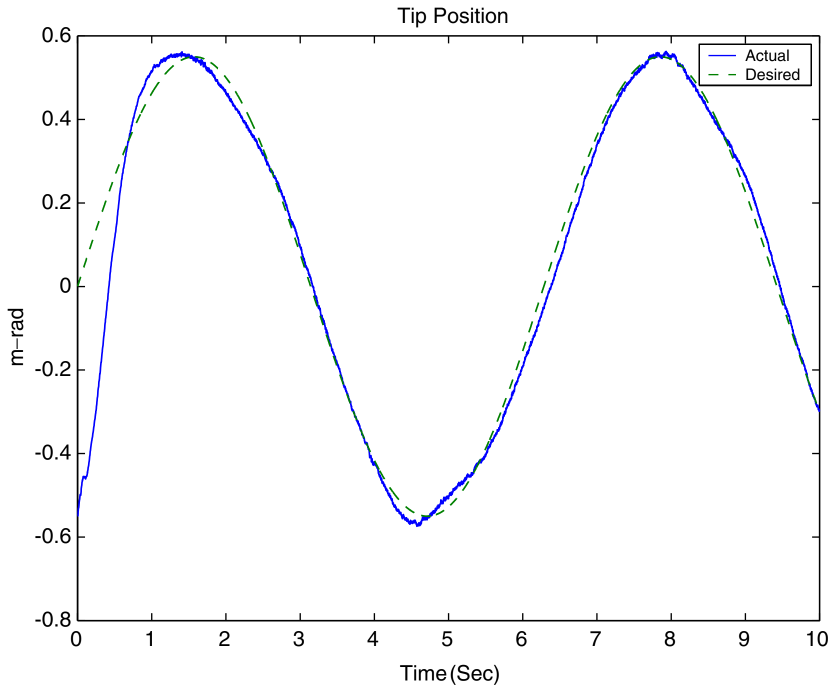

Fig. 7. Output tracking performance in experiment.

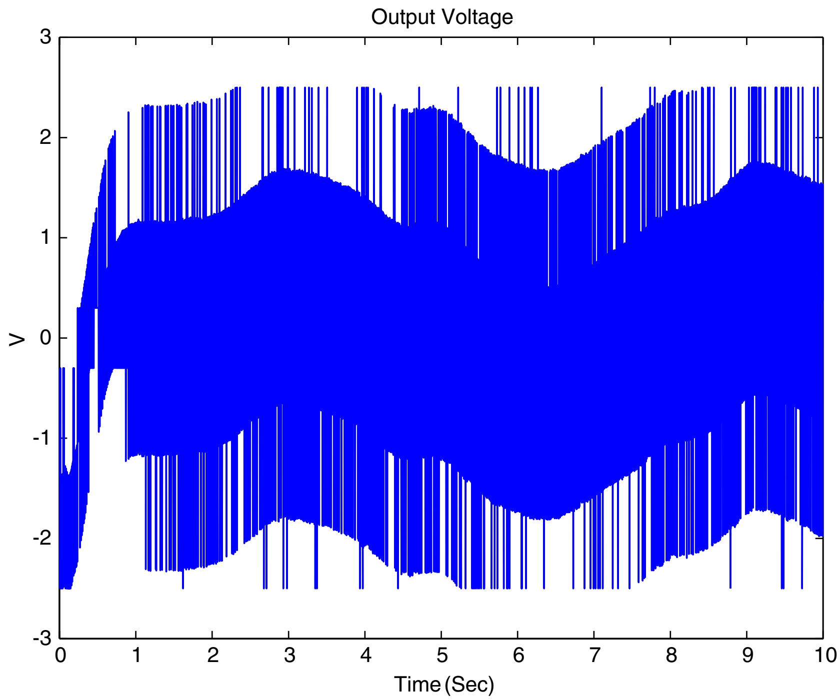

Fig. 8. Control voltage in experiment.

The first element in $\hat{\mathbf{w}}_g(0)$ is selected as a nonzero value to avoid singularity of $\hat{g}(0)$. The adaptive gain matrices are selected as

$$\mathbf{Q}_f = \mathrm{diag}(q_{f_1}, q_{f_2}, \ldots, q_{f_9}), \quad q_{f_i} = 10^{-3} \quad \forall i = 1, \ldots, 9,$$
$$\mathbf{Q}_g = \mathrm{diag}(q_{g_1}, q_{g_2}, \ldots, q_{g_9}), \quad q_{g_i} = 1 \quad \forall i = 1, \ldots, 9.$$

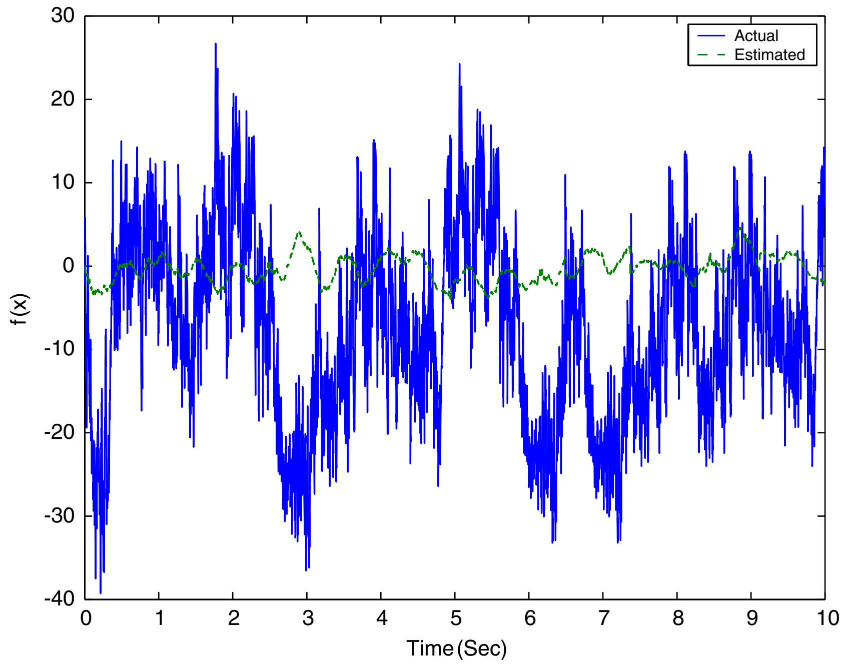

Fig. 9. Approximation of $f(x)$ in experiment.

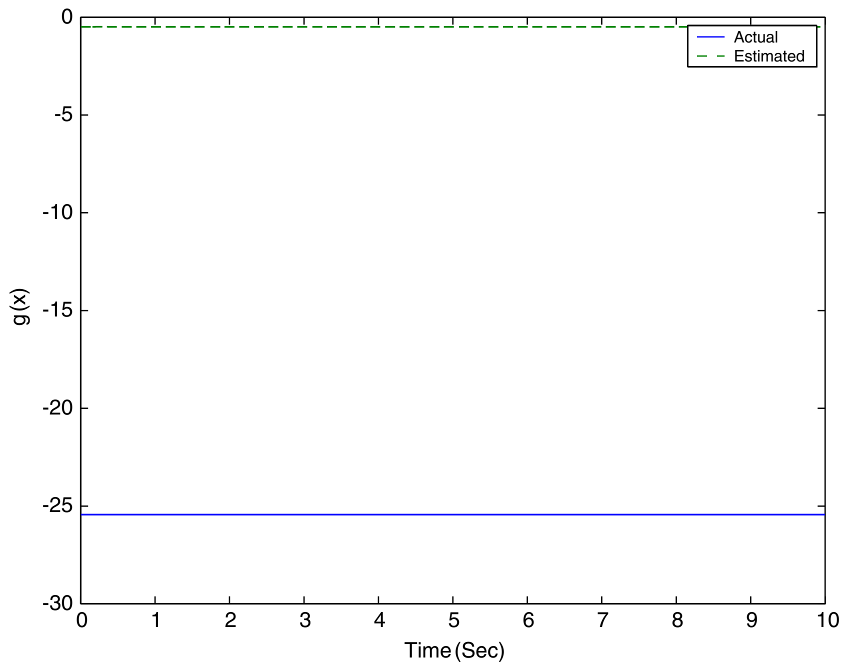

Fig. 10. Approximation of $g(x)$ in experiment.

Although other values of q_{f_i} and q_{g_i} are also possible to have output error convergence as long as $\mathbf{Q}_f$ and $\mathbf{Q}_g$ are positive definite, the above selection gives satisfactory performance. The simulation results are shown in Figs. 3–6. Fig. 3 shows that the end-point trajectory converges nicely to the desired output. It is noted that the output $y(x,t)$ defined in Eq. (1) is represented as the arc length; therefore, the unit of the vertical axis is m rad. Fig. 4 is the control torque in N m. Its peak value is about 20 N m in the transient. Figs. 5 and 6 present the estimation performance of f and g, respectively. Since the output error converges within 0.5 s, there is no enough driving force for the update law to have parameter convergence, which can also be observed from Eqs. (3) and (12). Although estimates of f and g do not converge to their true values, they remain bounded as proved in Eq. (13).

4.3. Experimental results

Same conditions are used in the real-time control on the actual flexible arm. The first nine terms of the Fourier series are still used to approximate both f and g, respectively. The experimental results are shown in Figs. 7–10. Fig. 7 shows the output tracking performance. Fig. 8 presents the voltage output of the D/A converter. Figs. 9 and 10 show the boundedness of the estimates of f and g, respectively. It is worth to note that in designing the controller we do not need much knowledge for the system. All we have to do is to pick some controller parameters and set some initial conditions. As for the number of terms of the basis functions, we firstly try an arbitrary number. If the performance is satisfactory, the number is reduced; otherwise, it is increased. After several times of adjustments, the number can be determined based on some compromise between computation efficiency and tracking performance. Therefore, the proposed controller is very easy to implement. As long as proper numbers of the basis functions are selected, the output error is proved to be convergent. To have better performance, some controller parameters can be adjusted.

5. Conclusion

We have proposed an adaptive controller for a single-link flexible arm based on FAT. Analysis of the closed-loop stability has been investigated with consideration of the approximation error. In implementing the controller, the control strategy does not need much knowledge about the system model as long as proper sets of basis functions are used. Both simulation and experimental results show that the proposed controller is able to give satisfactory tracking performance regardless of various uncertainties.

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